# Four-quark Operators Relevant to B Meson Lifetimes from QCD Sum Rules

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### Abstract

At the order of  $1/m_b^3$ , the B meson lifetimes are controlled by the hadronic matrix elements of some four-quark operators. The nonfactorizable magnitudes of these four-quark operator matrix elements are analyzed by QCD sum rules in the framework of heavy quark effective theory. The vacuum saturation for color-singlet four-quark operators is justified at hadronic scale, and the nonfactorizable effect is at a few percent level. However for color-octet four-quark operators, the vacuum saturation is violated sizably that the nonfactorizable effect cannot be neglected for the B meson lifetimes. The implication to the extraction of some of the parameters from B decays is discussed. The B meson lifetime ratio is predicted as  $\tau(B^-)/\tau(B^0) = 1.09 \pm 0.02$ . However, the experimental result of the lifetime ratio  $\tau(\Lambda_b)/\tau(B^0)$  still cannot be explained.

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Heavy hadron lifetimes provide us with testing ground to the Standard Model, especially to QCD in some aspects [1-3], because they can be systematically calculated within the framework of heavy quark expansion. Theoretically, if we do not assume the failure of the local duality assumption, the heavy hadron lifetime differences appear, at most, at the order of  $1/m_Q^2$  [4]. Recent experimental results on the lifetime ratio of  $\Lambda_b$  baryon and B meson [5] showed some deviation from the theoretical expectation. This has drawn a lot theoretical attentions [6-11]. The current experimental values for the lifetime ratios which we are interested in are [5]

$$\frac{\tau(B^{-})}{\tau(B^{0})} = 1.06 \pm 0.04 , 
\frac{\tau(\Lambda_{b})}{\tau(B^{0})} = 0.79 \pm 0.06 .$$
(1)

This may imply that the  $O(\frac{1}{m_b^2})$  contribution is not enough for the explanation of above heavy baryon and heavy meson lifetime difference. To the order of  $1/m_b^3$ , the hadron lifetimes have been studied since mid-80s [11, 12, 4, 6, 7]. And the potential importance of the  $1/m_b^3$  corrections has been pointed out. The parameterization of the hadronic matrix elements of four-quark operators which appear in the hadron lifetimes at the order of  $1/m_b^3$  is generally expressed as [7]

$$\langle \bar{B}|\bar{b}\gamma_{\mu}(1-\gamma_{5})q\bar{q}\gamma^{\mu}(1-\gamma_{5})b|\bar{B}\rangle \equiv B_{1}F_{B}^{2}m_{B}^{2},$$

$$\langle \bar{B}|\bar{b}(1-\gamma_{5})q\bar{q}(1+\gamma_{5})b|\bar{B}\rangle \equiv B_{2}F_{B}^{2}m_{B}^{2},$$

$$\langle \bar{B}|\bar{b}\gamma_{\mu}(1-\gamma_{5})t_{a}q\bar{q}\gamma^{\mu}(1-\gamma_{5})t_{a}b|\bar{B}\rangle \equiv \epsilon_{1}F_{B}^{2}m_{B}^{2},$$

$$\langle \bar{B}|\bar{b}(1-\gamma_{5})t_{a}q\bar{q}(1+\gamma_{5})t_{a}b|\bar{B}\rangle \equiv \epsilon_{2}F_{B}^{2}m_{B}^{2},$$

$$(2)$$

and

$$\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) b | \Lambda_b \rangle \equiv -\frac{F_B^2 m_B}{12} r , 
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | \bar{b} (1 - \gamma_5) q \bar{q} (1 + \gamma_5) b | \Lambda_b \rangle \equiv -\tilde{B} \frac{F_B^2 m_B}{24} r ,$$
(3)

where the parameters  $B_i$ ,  $\epsilon_i$  (i = 1, 2),  $F_B$ , r and  $\tilde{B}$  should be calculated by some nonperturbative QCD method. In above equations, the renormalization scale is arbitrary, and the parameters depend on it. It can be taken naturally at the low hadronic scale to apply heavy quark expansion. On the other hand, if the scale is taken at  $m_b$ ,

parameter  $F_B(m_b)$  is just the well-defined measurable physical quantity – B meson decay constant  $f_B$ .

The QCD sum rule [13], which is regarded as a nonperturbative method rooted in QCD itself, has been used successfully to calculate the properties of various hadrons. In Ref. [8], in the framework of heavy quark effective theory (HQET), the baryonic parameters r and  $\tilde{B}$  have been calculated by QCD sum rule,  $r \sim 0.1 - 0.3$ ,  $\tilde{B} \simeq 1$ . For a complete analysis, the mesonic parameters  $B_i$  and  $\epsilon_i$  should be also calculated from QCD sum rule. The four-quark operators, and hence  $B_i$ ,  $\epsilon_i$ , r and  $\tilde{B}$ , are scale-dependent quantities when the QCD radiative corrections are included. It was proposed by Shifman and Voloshin [11] that at the low hadronic scale, the vacuum saturation approximation, namely  $B_i = 1$  and  $\epsilon_i = 0$ , makes sense. However in this case, the measured lifetime ratio  $\tau(\Lambda_b)/\tau(B^0)$  cannot be explained [8]. There are some argument, on the other hand, that the vacuum saturation maybe a poor approximation [9]. Especially from a naive large  $N_c$  analysis,  $\epsilon_i$ 's are about  $1/N_c \sim 0.3$  [11, 7]. We will explore the violation of the vacuum saturation approximation in detail from QCD sum rules in the framework of HQET.

Let us first consider the parameters  $B_i$ . We construct the following three-point Green's function,

$$\Gamma^{O}(\omega,\omega') = i^{2} \int dx dy e^{ik'\cdot x - ik\cdot y} \langle 0|\mathcal{T}[\bar{q}(x)\gamma^{\mu}\gamma_{5}h_{v}^{(b)}(x)]O(0)[\bar{q}(y)\gamma_{\mu}\gamma_{5}h_{v}^{(b)}(y)]^{\dagger}|0\rangle , \quad (4)$$

where  $\omega = 2v \cdot k$ ,  $\omega' = 2v \cdot k'$ ;  $h_v^{(b)}$  is the b-quark field in the HQET with velocity v. And O denotes the color-singlet operators given in Eq. (2),

$$O = \bar{b}\Gamma_1 q\bar{q}\Gamma_2 b , \qquad (5)$$

with  $\Gamma_1 = \Gamma_2 = \gamma^{\mu}(1 - \gamma_5)$  for  $B_1$  and  $\Gamma_1 = 1 - \gamma_5$ ,  $\Gamma_2 = 1 + \gamma_5$  for  $B_2$ . In terms of the hadronic expression, the parameter  $B_i$  appears in the ground state contribution of  $\Gamma^O(\omega, \omega')$ ,

$$\Gamma^{O}(\omega, \omega') = B_{i} \frac{F_{B}^{4} m_{B}^{2}}{(2\bar{\Lambda} - \omega)(2\bar{\Lambda} - \omega')} + \text{resonances} , \qquad (6)$$

where  $\bar{\Lambda} = m_B - m_b$ . The resonance contribution will be simulated by the perturbative QCD contribution above some threshold energy due to the local duality assumption. On the other hand, this Green's function  $\Gamma^O(\omega, \omega')$  will be calculated in terms of quarks and gluons, that is to say, by the operator product expansion method of QCD. The essential feature of the QCD sum rule is that in the QCD calculation, the vacuum condensates of quarks and gluons have to be included. Practically the calculation is performed at 1 GeV scale or so, only a few terms of the condensates with lowest dimensions are important. Note that this calculation will be reinforced by the Borel transformation, and its consistency should be checked through the finding of the so-called sum rule window. This procedure will be explained in more detail in the calculation of parameters  $\epsilon_i$ .

The calculation of  $\Gamma^O(\omega,\omega')$  in HQET is straightforward. The fixed point gauge [14] is adopted. The assumption that the four-quark condensates are factorizable is used. The violation of it will be discussed later. The dominant non-vanishing Feynman diagrams are shown in Fig. 1 where the double lines denote the heavy quark. However, all these diagrams are factorizable, namely they do not contribute any deviation from vacuum saturation. And this is true even when the  $O(\alpha_s)$  radiative corrections to these diagrams are included, simply because O is a color-singlet operator. Note that due to the same reason, there is no nonfactorizable gluon condensate contribution. The tadpole diagrams in which the light quark lines from the four-quark vertex are contracted have been subtracted. Generally, the nonfactorizable diagrams for a color-singlet four-quark operator are listed in Fig. 2. They would be the leading contribution diagrams which might have an O(1) correction to  $B_i = 1$ . But it is easy to see that they are vanishing because of the special structure of  $\Gamma_1$  and  $\Gamma_2$  in O (Eq. (5)). Hence, we see that the vacuum saturation approximation is valid at hadronic scale for color-singlet operators,  $B_i = 1$ , for i = 1, 2, through the leading order consideration.

To what extent the nonfactorizable effect makes  $B_i$  deviate from unity? The next

possibility for non-factorization is to consider two gluon exchanges, like those in Fig. 3. It is interesting to note that the four-gluon condensate  $\langle \alpha_s^2 G^4 \rangle$  contribution to  $\Gamma^O(\omega, \omega')$ , like Fig. 3(b), is vanishing<sup>2</sup>. The perturbative two gluon exchange diagram, namely Fig. 3(a), can be the leading non-vanishing nonfactorizable contribution. (There are some other diagrams which have the same or less order of magnitude numerically, like Fig. 3(c).) This is a four-loop diagram, its contribution to  $B_i$  is estimated as  $(\frac{\alpha_s(1\text{GeV})}{\pi})^2$  which is numerically around a few percent and is too small to be important for the hadron lifetimes. We obtain that

$$B_i = 1 + O(10^{-2}), \qquad (i = 1, 2).$$
 (7)

The parameters  $\epsilon_i$  (i=1, 2), as we will see in the following analysis, have deviation from the expectation of the vacuum saturation at the hadronic scale which cannot be neglected for the B meson lifetimes. The procedure is similar to that for parameters  $B_i$ . The three-point Green's function in this case is constructed to be

$$\Gamma^{T}(\omega,\omega') = i^{2} \int dx dy e^{ik'\cdot x - ik\cdot y} \langle 0|\mathcal{T}[\bar{q}(x)\gamma^{\mu}\gamma_{5}h_{v}^{(b)}(x)]T(0)[\bar{q}(y)\gamma_{\mu}\gamma_{5}h_{v}^{(b)}(y)]^{\dagger}|0\rangle , \quad (8)$$

where T is the color-octet operators given in Eq. (2),

$$T = \bar{b}\Gamma_1 t_a q \bar{q} \Gamma_2 t_a b . (9)$$

In the hadronic language, the parameter  $\epsilon_i$  appears in the ground state contribution of  $\Gamma^T(\omega, \omega')$ ,

$$\Gamma^{T}(\omega, \omega') = \epsilon_{i} \frac{F_{B}^{4} m_{B}^{2}}{(2\bar{\Lambda} - \omega)(2\bar{\Lambda} - \omega')} + \text{resonances} .$$
 (10)

In the calculation of  $\Gamma^T(\omega, \omega')$ , all the condensates with dimensions lower than 6 are retained. The dominant diagrams are found to be those given in Fig. 4, and

<sup>&</sup>lt;sup>2</sup>This fact is gauge invariant and can be easily observed in the fixed point gauge. In this gauge, the light quark propagator with two condensate gluons attached vanishes if one end of the propagator is at space-time origin [14]. And in this gauge, the heavy quarks in Figs. 1-3 are free from interaction.

they are non-vanishing. The four-quark condensate  $\langle \bar{q}q \rangle^2$  diagram vanishes. We have neglected the perturbative diagrams which are three-loop diagrams of order  $\alpha_s$ . That means the QCD radiative corrections to  $\epsilon_i$  are not included. From general experience of QCD sum rule method, the condensate diagrams play more dominant role than the corresponding perturbative one. This neglect is expected viable. The condensates parameterize the non-perturbative effects which in the 1 GeV scale are still small enough to be treated as power corrections of  $1/\omega$  and  $1/\omega'$  in the operator product expansion. Because the perturbative contribution has been neglected, resonances in Eq. (10) will be also neglected due to the duality assumption. While the calculation can be justified if  $(-\omega)$  and  $(-\omega')$  are large, however the hadron ground state property should be obtained at small  $(-\omega)$  and  $(-\omega')$ . These contradictory requirements can be achieved by introducing double Borel transformation for  $\omega$  and  $\omega'$ . There are two Borel parameters  $\tilde{T}$  and  $\tilde{T}'$  corresponding to  $\omega$  and  $\omega'$ , respectively. They appear symmetrically, so we take  $\tilde{T} = \tilde{T}'$  in the analysis.

The sum rules for the parameters  $\epsilon_i$  are

$$\epsilon_{1} = m_{0}^{2} \langle \bar{q}q \rangle \frac{\tilde{T}}{32\pi^{2}} \frac{e^{4\bar{\Lambda}/\tilde{T}}}{F_{B}^{4}m_{B}^{2}} ,$$

$$\epsilon_{2} = -\left(\frac{\langle \alpha_{s}GG \rangle}{4\pi} \tilde{T} + m_{0}^{2} \langle \bar{q}q \rangle\right) \frac{\tilde{T}}{16\pi^{2}} \frac{e^{4\bar{\Lambda}/\tilde{T}}}{F_{B}^{4}m_{B}^{2}} , \tag{11}$$

There is no gluon condensate contribution to  $\epsilon_1$ . Numerically we use the following values of the condensates,

$$\langle \bar{q}q \rangle \simeq -(0.23 \text{ GeV})^3 ,$$
  
 $\langle \alpha_s GG \rangle \simeq 0.04 \text{ GeV}^4 ,$  (12)  
 $\langle g\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \rangle \equiv m_0^2 \langle \bar{q}q \rangle , \quad m_0^2 \simeq 0.8 \text{ GeV}^2 .$ 

For consistence, the HQET sum rule of the parameter  $F_B$  [15, 16] will be used. The result derived in Ref. [16] is

$$F_B^2 m_B e^{-2\bar{\Lambda}/\tilde{T}} = \frac{3}{8\pi^2} \int_0^{\omega_c} d\nu \nu^2 e^{-\nu/\tilde{T}} - \langle \bar{q}q \rangle \left( 1 - \frac{m_0^2}{4\tilde{T}^2} \right) , \qquad (13)$$

where  $\omega_c$  is twice of the continuum threshold, which was determined as  $\omega_c \simeq 2.0 \pm 0.3$  GeV. The range of the Borel parameter  $\tilde{T}$  in Eq. (11) should be similar to that of  $F_B$ . This point can be obviously seen from the sum rules for  $B_i$  if it is written down explicitly. In that sum rules, if all the non-factorizable contributions are neglected, then the resulting  $B_i$  is unity only if the Borel parameter is the same as that of  $F_B$ . Practically we take the window as  $0.7 \leq \tilde{T} \leq 1.0$  GeV. There is no  $\bar{\Lambda}$  dependences for  $\epsilon_i$ 's, because they are canceled actually. The numerical sum rule results for  $\epsilon_1$  and  $\epsilon_2$  are given in Fig. 5. From the figures, we see that  $\epsilon_i$ 's have no good stability in the window of  $\tilde{T}$ . This is because we have not included the perturbative diagram. Finally the results for  $\epsilon_1$  and  $\epsilon_2$  are obtained as

$$\epsilon_1 \simeq -(4.1 \pm 2.2) \times 10^{-2} ,$$

$$\epsilon_2 \simeq (6.1 \pm 3.5) \times 10^{-2} .$$
(14)

In spite of the large uncertainties in above equation, the numbers are significant for the B meson lifetime difference. We see that the vacuum saturation approximation for the color-octet matrix elements is indeed violated. The magnitudes of  $\epsilon_i$  can be as large as 0.1.

It is necessary to discuss the hypothesis on the four-quark condensate factorization. We have used it in above analysis for  $B_i$  and  $\epsilon_i$ . While this is the working assumption for the usual QCD sum rule calculations, the violation of it may imply in our case, new contribution to the non-factorizable effect of the four-quark operator matrix elements. This contribution to  $B_i$  and  $\epsilon_i$  is estimated as

$$\delta B_i \simeq \delta \epsilon_i = \delta \langle \bar{q}q \rangle^2 \frac{e^{4\bar{\Lambda}/\tilde{T}}}{F_B^4 m_B^2} ,$$
 (15)

where  $\delta \langle \bar{q}q \rangle^2$  denotes the deviation of the four-quark condensates from factorization. Arguments based on large  $N_c$  expansion suggest that this approximation is good to within  $1/N_c^2 \sim 10\%$  [17]. Numerically, considering the success of this assumption in the calculation of baryons by QCD sum rules [18], we take  $\delta \langle \bar{q}q \rangle^2$  to be less than 30% of  $\langle \bar{q}q \rangle^2$ . In this case,  $\delta B_i$  and  $\delta \epsilon_i$  are smaller than about  $10^{-2}$  which have no significant influence on our results for  $B_i$  and  $\epsilon_i$ .

It should be noted that the vacuum saturation and its violation we have analyzed above are at some hadronic scale, other than the scale of  $m_b$ . Because we have been working in the framework of HQET, in which the natural scale is  $\mu_{had} \ll m_b$ . The renormalization group evolution of the relevant operators was calculated in Refs. [19, 7]. Information on parameters  $B_i$  and  $\epsilon_i$  at scale  $m_b$  is necessary to obtain the hadron lifetimes by using the analysis of Ref. [7]. There is a notation difference in this paper. In HQET,  $F_B$  given in Eq. (13) is in fact a scale-dependent quantity if the renormalization effect is considered. From this point of view,  $F_B$  in our previous analysis took value at hadronic scale, whereas that in Ref. [7] at scale of  $m_b$ . Taking this difference into consideration, we have the following evolution relations by choosing  $\alpha_s(\mu_{had}) = 0.5$  (corresponding to  $\mu_{had} \sim 0.67$  GeV),

$$B_i(m_b) \simeq B_i(\mu_{had}) - 0.24\epsilon_i(\mu_{had}) ,$$

$$\epsilon_i(m_b) \simeq -0.05B_i(\mu_{had}) + 0.72\epsilon_i(\mu_{had}) .$$
(16)

That is

$$B_1(m_b) \simeq 1.01 \pm 0.01 , \qquad B_2(m_b) \simeq 0.99 \pm 0.01 ,$$
  
 $\epsilon_1(m_b) \simeq -0.08 \pm 0.02 , \qquad \epsilon_2(m_b) \simeq -0.01 \pm 0.03 ,$ 

$$(17)$$

from Eq. (14).

To be more relevant, we discuss a more detailed parameterization for the fourquark operators proposed in Ref. [6] in the following. It is motivated for the modelindependent determination of the hadronic matrix elements from the lepton spectrum in the endpoint region of semileptonic B decays. The parameterization is written as

$$\langle \bar{B}|\bar{b}\gamma_{\mu}(1-\gamma_{5})q\bar{q}\gamma_{\nu}(1-\gamma_{5})b|\bar{B}\rangle \equiv (v_{singl}v_{\mu}v_{\nu}-g_{singl}g_{\mu\nu})F_{B}^{2}m_{B}^{2},$$
  
$$\langle \bar{B}|\bar{b}\gamma_{\mu}(1-\gamma_{5})t_{a}q\bar{q}\gamma_{\nu}(1-\gamma_{5})t_{a}b|\bar{B}\rangle \equiv (v_{oct}v_{\mu}v_{\nu}-g_{oct}g_{\mu\nu})F_{B}^{2}m_{B}^{2},$$
(18)

where  $v_{singl}$ ,  $g_{singl}$  and  $v_{oct}$ ,  $g_{oct}$  are the parameters. The tadpole diagram contributions of the matrix elements have been subtracted. The parameters are related to those of Eq. (2) as  $v_{singl} - 4g_{singl} = B_1$  and  $v_{oct} - 4g_{oct} = \epsilon_1$ . In the vacuum saturation approximation,  $v_{singl} = 1$ ,  $g_{singl} = v_{oct} = g_{oct} = 0$ . They can be calculated by QCD sum rules. From our above analysis for the color-singlet operator, we have

$$v_{singl} \simeq 1 + O(10^{-2}), \quad g_{singl} \simeq O(10^{-2}).$$
 (19)

The sum rules for the color-octet operator are obtained as

$$v_{oct} = -\left(\frac{\langle \alpha_s GG \rangle}{6\pi} \tilde{T} + m_0^2 \langle \bar{q}q \rangle\right) \frac{\tilde{T}}{8\pi^2} \frac{e^{4\bar{\Lambda}/\tilde{T}}}{F_B^4 m_B^2} ,$$

$$g_{oct} = -\left(\frac{\langle \alpha_s GG \rangle}{12\pi} \tilde{T} + m_0^2 \langle \bar{q}q \rangle\right) \frac{\tilde{T}}{16\pi^2} \frac{e^{4\bar{\Lambda}/\tilde{T}}}{F_B^4 m_B^2} .$$
(20)

The numerical results for  $v_{oct}$  and  $g_{oct}$  are given in Fig. 6 from which we obtain

$$v_{oct} \simeq (1.35 \pm 0.76) \times 10^{-1} , \qquad g_{oct} \simeq (0.74 \pm 0.40) \times 10^{-1} .$$
 (21)

They are typically at the order of 0.1.

From the QCD sum rule point of view, the vacuum saturation for the hadronic matrix elements of the color-singlet four-quark operators given in Eq. (5) or in the first equation of (18) is valid up to a few percent level. However, that for the color-octet four-quark operators is violated at ten percent level. The result that the nonfactorizable effect for color-octet operators is more significant can be also understood qualitatively, if we look at the perturbative diagrams. This effect is at three-loop level, whereas that for color-singlet operators at the four-loop level.

The  $1/m_b$  corrections to above results can be analyzed in principle. While having little influence on our above arguments and calculations, they formally belong to the  $O(1/m_b^4)$  effects to the b-hadron lifetimes.

In Ref. [6], the extraction of the parameters from B decays are discussed. The role

of parameter  $g_{singl}$  might be significant in comparing the decay rates of  $B^0 \to l\nu X_u$  and  $B^- \to l\nu X_u$ . From our analysis, the difference of the decay rates should be small due to  $g_{singl}$  is very small even at the scale of  $m_b$ . The ratio of  $g_{singl}/g_{oct}$  can be obtained from the lepton spectrum of the above decays. While having not determined the sign of  $g_{singl}$ , our analysis prefers an even smaller ratio of  $|g_{singl}/g_{oct}| \sim 1/10$  than Ref. [6] did. These points will be verified by the experiments in the near future.

The b-hadron lifetime ratios are expressed as [7]

$$\frac{\tau(B^{-})}{\tau(B^{0})} = 1 + 0.03B_{1} - 0.71\epsilon_{1} + 0.20\epsilon_{2} + O(\frac{1}{m_{b}^{4}}), 
\frac{\tau(\Lambda_{b})}{\tau(B^{0})} = 0.98 - 0.17\epsilon_{1} + 0.20\epsilon_{2} - (0.013 + 0.022\tilde{B})r + O(\frac{1}{m_{b}^{4}}).$$
(22)

In above equation, the energy scale for the parameters is at  $m_b$ . the B meson decay constant  $f_B$  is taken as 200 MeV. By using the results in Eq. (17), we obtain

$$\frac{\tau(B^-)}{\tau(B^0)} \simeq 1.09 \pm 0.02$$
 and  $\frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.98 \pm 0.01$ . (23)

In obtaining above numbers, the QCD sum rule results for the baryonic parameters r and  $\tilde{B}$  [8] have been also taken values at scale  $m_b$ . From Eq. (23) we see that, while  $\tau(B^-)/\tau(B^0)$  is agree with measurement, the QCD sum rule result for  $\tau(\Lambda_b)/\tau(B^0)$ , after including  $1/m_b^3$  corrections, still contradicts the experimental measurement.

In summary, the nonfactorizable contributions of the hadronic matrix elements of four-quark operators relevant to the B meson lifetimes have been studied by QCD sum rules in the framework of HQET. The vacuum saturation for color-singlet four-quark operators is justified at hadronic scale, and the nonfactorizable effect is at a few percent level. However, the vacuum saturation for color-octet four-quark operators is violated sizably that the nonfactorizable effect cannot be neglected for the B meson lifetimes. The implication to the extraction of some of the parameters from B decays has been discussed. The B meson lifetime ratio has been predicted to be consistent with experiment,  $\tau(B^-)/\tau(B^0) = 1.09 \pm 0.02$ . However, the discrepancy between theory

and experiment on  $\tau(\Lambda_b)/\tau(B^0)$  has not been solved. It is unlikely that higher  $1/m_b$  corrections will give the solution. If the experimental data of  $\tau(\Lambda_b)/\tau(B^0)$  is further confirmed in the future, that may imply either the failure of local duality assumption or some new physics.

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## Figure captions

- Fig. 1. Dominant non-vanishing Feynman diagrams for  $\Gamma^{O}(\omega, \omega')$ .
- Fig. 2. Nonfactorizable diagrams for a general color-singlet four-quark operator.
- Fig. 3. Nonfactorizable diagrams by two gluon exchanges for  $\Gamma^{O}(\omega, \omega')$ .
- Fig. 4. Condensate contribution to  $\Gamma^T(\omega, \omega')$ .
- Fig. 5. Sum rules for  $\epsilon_1$  (a) and  $\epsilon_2$  (b). The sum rule window is  $\tilde{T}=0.7-1.0$  GeV.
- Fig. 6. Sum rules for  $v_{oct}$  (a) and  $g_{oct}$  (b).

# Figures

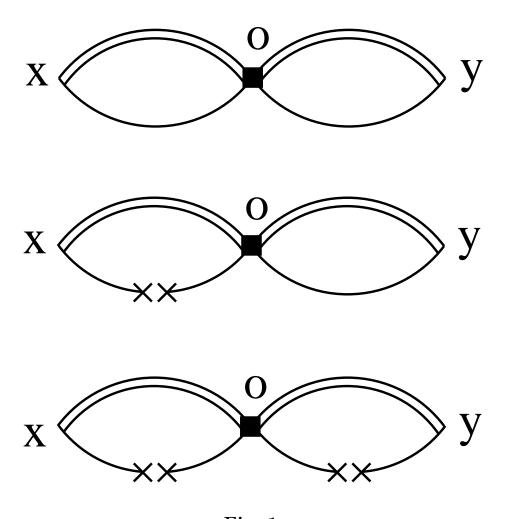


Fig. 1

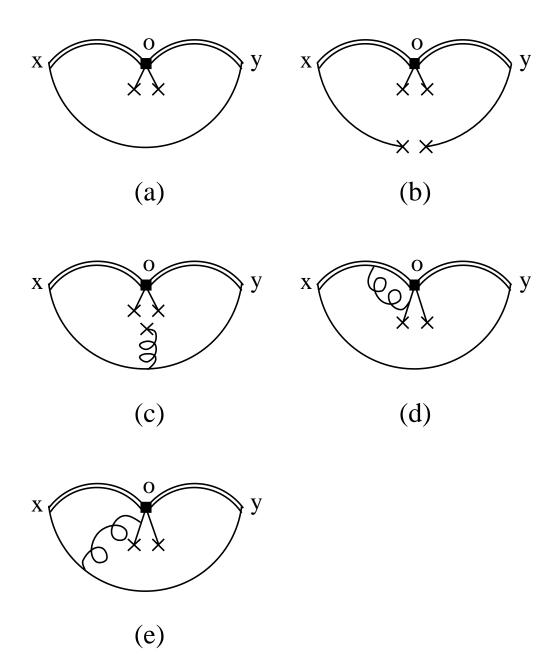
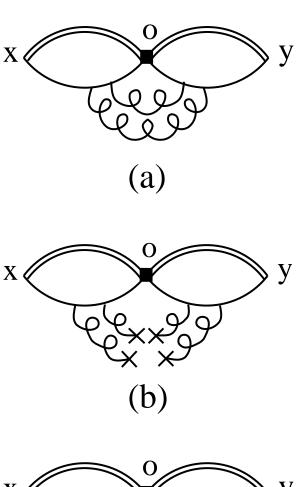


Fig. 2



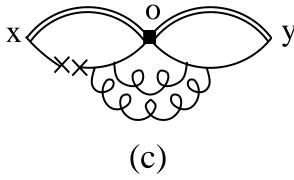
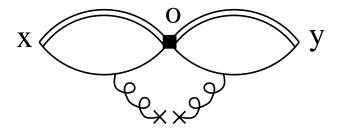


Fig. 3



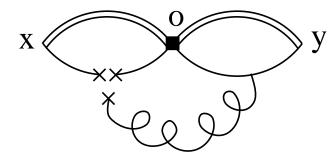


Fig. 4

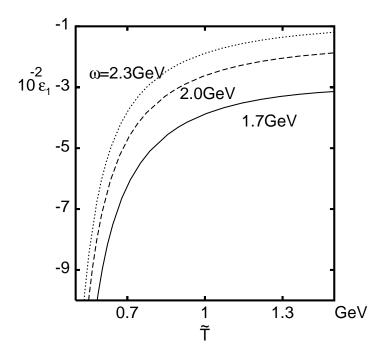


Fig. 5(a)

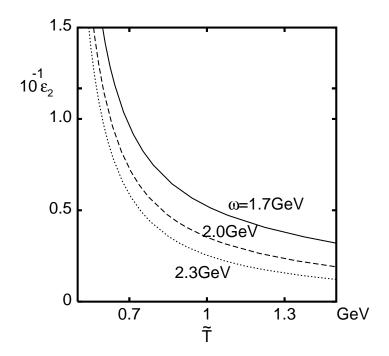


Fig. 5(b)

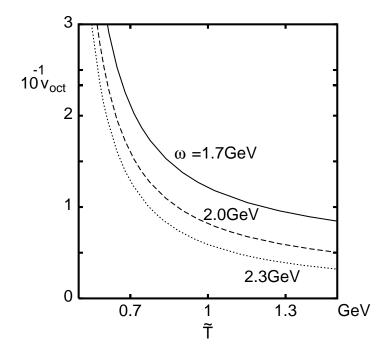


Fig. 6(a)

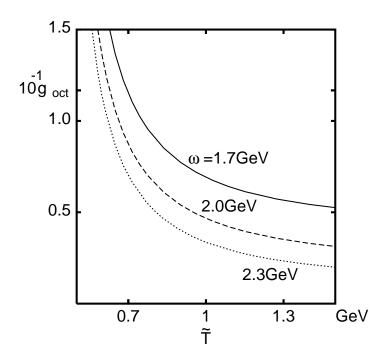


Fig. 6(b)